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Pair creation: back-reactions and damping

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Abstract

We solve the quantum Vlasov equation for fermions and bosons, incorporating spontaneous pair creation in the presence of back-reactions and collisions. Pair creation is initiated by an external impulse field and the source term is non-Markovian. A simultaneous solution of Maxwell's equation in the presence of feedback yields an internal current and electric field that exhibit plasma oscillations with a period τ_{pl}. Allowing for collisions, these oscillations are damped on a time-scale, τ_r, determined by the collision frequency. Plasma oscillations cannot affect the early stages of the formation of a quark-gluon plasma unless $\tau_r \gg \tau_{pl}$ and $\tau_{pl} \sim 1/\Lambda_{\rm QCD} \sim 1\,{\rm fm/c}$.

I. INTRODUCTION

The nonperturbative Schwinger mechanism [1], which describes the spontaneous formation of fermion-antifermion pairs, has been used [2] to model the formation of a quark-gluon plasma in heavy ion collisions. In this approach [3] nucleon-nucleon collisions lead to the creation of flux-tubes, in which quark-antiquark pairs are connected by a strong colour-electric field. The energy density (string tension) acts like a strong background field and particle-antiparticle pairs are created via the Schwinger mechanism [4–7]. These charged particles polarise the vacuum and are accelerated in the external field. Their motion generates a field that in turn modifies the initial background field and, in the absence of further interactions, that back-reaction induces plasma oscillations.

The back-reaction phenomenon has become a focus of attention in recent years, both in general and as it can arise in the pre-equilibrium stage of an heavy ion collision. Theoretical approaches as diverse as field theory [8–10] and transport equations [11,12] have been applied. The link between treatments based on the field equations and the formulation of a Boltzmann equation was recently investigated [13,14]. These studies show that the resulting kinetic equation has a non-Markovian source term. For weak fields there is no overlap between the time-scales characterising vacuum tunnelling and the period between pair production events: τ_{qu}, τ_{prod}, and the Markovian approximation to the quantum Vlasov equation is valid [14].

However, for strong background fields there is an overlap between these time-scales and this makes the non-Markovian nature of the source term very important [15–17]. Back-reactions and collisions introduce at least two more time-scales: the plasma oscillation period, τ_{pl}, and the collision period, τ_r, and their impact is an integral focus of this article. Furthermore, in contrast to other recent studies [11,14], we induce particle production by a time-dependent external field.

In Sect. II we review the main equations and results for particle creation using a non-Markovian source term. In Sect. III we derive the renormalised Maxwell equation determined by the external and internal fields and, for special choices of the external field, present numerical results obtained by solving the coupled system of kinetic and Maxwell equations for bosons and fermions, with and without a simple collision term. We summarise our results in Sect. IV.

II. PAIR CREATION WITH A NON-MARKOVIAN SOURCE TERM

We consider an external, spatially-homogeneous, time-dependent vector potential A_μ, in Coulomb gauge: $A_0 = 0$, and write $\vec{A} = (0, 0, A(t))$. The corresponding electric field is

$$E(t) = -\dot{A}(t) := -dA(t)/dt \,. \tag{1}$$

The kinetic equation satisfied by the single-particle distribution function: $f_\pm$ ("+" for bosons, "−" for fermions) is

$$\frac{df_\pm(\vec{p},t)}{dt} = S_\pm(\vec{p},t), \tag{2}$$

where the source term is momentum- and time-dependent:

$$S_\pm(\vec{p},t) = \frac{1}{2}\mathcal{W}_\pm(t)\int_{-\infty}^{t} dt'\mathcal{W}_\pm(t')F_\pm(\vec{p},t)\cos[x(t',t)]\,, \tag{3}$$

with $x(t',t) := 2[\Theta(t) - \Theta(t')]$ describing the difference between the dynamical phases

$$\Theta(t) = \int_{-\infty}^{t} dt'\omega(t') \ . \tag{4}$$

Here the total energy is

$$\omega(t) = \sqrt{\varepsilon_\perp^2 + P_\parallel^2(t)} \tag{5}$$

where $\varepsilon_\perp = \sqrt{m^2 + \vec{p}_\perp^{\,2}}$ is the transverse energy and we have introduced the kinetic momentum: $\vec{P} = (p_\perp, P_\parallel(t))$, with $\vec{p}_\perp = (p_1, p_2)$, $P_\parallel(t) = p_\parallel - eA(t)$.

Equation (2) was recently derived from the underlying quantum field theory [13–15] and exhibits a number of interesting new features. For example, in a strong background field its solutions describe an enhancement in the boson production rate and a suppression of fermion production. There are two aspects [16,17] of Eq. (2) that generate such differences between the solutions for fermions and bosons: the different transition coefficients

$$\mathcal{W}_\pm(t) = \frac{eE(t)P_\parallel(t)}{\omega^2(t)}\Big(\frac{\varepsilon_\perp}{P_\parallel(t)}\Big)^{g_\pm - 1}\,, \tag{6}$$

where the degeneracy factor is $g_+ = 1$ for bosons and $g_- = 2$ for fermions; and the statistical factor: $F_\pm(\vec{p},t) = [1 \pm 2f_\pm(\vec{p},t)]$.

The kinetic equation, Eq. (2), is non-Markovian for two reasons: (i) the source term on the right-hand-side (r.h.s) requires knowledge of the entire history of the evolution of the distribution function, from $t_{-\infty} \to t$; and (ii), even in the low density limit ($F(t) = 1$), the integrand is a nonlocal function of time as is apparent in the coherent phase oscillation term: $\cos[x(t',t)]$. The mean field approaches of Refs. [8–10] also incorporate non-Markovian effects in particle production. However, the merit of a kinetic formulation lies in the ability to make a simple and direct connection with widely used approximations.

In the low density limit the source term is independent of the distribution function

$$S^0_\pm(\vec{p},t) = \frac{1}{2}\mathcal{W}_\pm(t)\int_{-\infty}^t dt'\mathcal{W}_\pm(t')\cos[x(t',t)] \tag{7}$$

and Eq. (2) becomes

$$\frac{df^0_\pm(\vec{p},t)}{dt} = S^0_\pm(\vec{p},t)\,. \tag{8}$$

(The low-density limit can only be self-consistent for weak fields.) Even in this case there are differences between the solutions for fermions and bosons because of the different coefficients $\mathcal{W}_\pm(t)$, and the equation remains nonlocal in time. Equation (8) has the general solution

$$f^0_\pm(\vec{p},t) = \int_{-\infty}^t dt' S^0_\pm(\vec{p},t'), \tag{9}$$

which provides an excellent approximation to the solution of the complete equation when the background field strength is small compared to the transverse energy.

The ideal Markov limit was found in Ref. [14], where a further asymptotic expansion was employed and a **local** source term for weak electric fields was derived. In this case $\tau_{qu} < \tau_{prod}$. However, for very strong fields a clear separation of these time-scales is not possible and the kinetic equation must be solved in its non-Markovian form where memory effects are important [17].

Equation (2) is an integro-differential equation. It can be re-expressed by introducing

$$v_\pm(\vec{p},t) = \int_{t_0}^t dt'\,\mathcal{W}_\pm(\vec{p},t')F_\pm(\vec{p},t')\,\cos[x(\vec{p},t,t')]\,, \tag{10}$$

$$z_\pm(\vec{p},t) = \int_{t_0}^t dt'\,\mathcal{W}_\pm(\vec{p},t')F_\pm(\vec{p},t')\,\sin[x(\vec{p},t,t')]\,, \tag{11}$$

in which case we have

$$\frac{\partial f_\pm(\vec{P},t)}{\partial t} + eE(t)\frac{\partial f_\pm(\vec{P},t)}{\partial P_\parallel} = \tfrac{1}{2}\mathcal{W}_\pm(\vec{P},t')v_\pm(\vec{P},t), \tag{12}$$

$$\frac{\partial v_\pm(\vec{P},t)}{\partial t} + eE(t)\,\frac{\partial v_\pm(\vec{P},t)}{\partial P_\parallel} = \mathcal{W}_\pm(\vec{P},t')F_\pm(\vec{P},t) - 2\omega(\vec{P})z_\pm(\vec{P},t)\,, \tag{13}$$

$$\frac{\partial z_\pm(\vec{P},t)}{\partial t} + eE(t)\frac{\partial z_\pm(\vec{P},t)}{\partial P_\parallel} = 2\omega(\vec{P})v_\pm(\vec{P},t)\,, \tag{14}$$

with the initial conditions $f_\pm(t_0) = v_\pm(t_0) = z_\pm(t_0) = 0$, where $t_0 \to -\infty$. This coupled system of linear differential equations is much simpler to solve numerically.

III. BACK-REACTIONS

A. The Maxwell equation

In recent years the effect of back-reactions in inflationary cosmology and also in the evolution of a quark-gluon plasma has been studied extensively. In both cases the particles produced by the strong background field modify that field: in cosmology it is the time-dependent gravitational field, which couples via the masses, and in a quark-gluon plasma, it is the chromoelectric field affected by the partons' colour charge.

In our previous studies we have considered constant [17] and simply-constructed time-dependent [15] Abelian electric fields but ignored the effect of back-reactions; i.e., that the particles produced by the background field are accelerated by that field, generating a current that opposes and weakens it, and can also lead [8,9] to plasma oscillations. The effect of this back-reaction on the induced field is accounted for by solving Maxwell's equation: $\dot{E}(t) = -j(t)$. Herein we assume that the plasma is initially produced by an external field, $E_{ex}(t)$, excited by an external current, $j_{ex}(t)$, such as might represent a heavy ion collision and this is our model-dependent input. The total field is the sum of that external field and an internal field, $E_{in}(t)$, generated by the internal current, $j_{in}(t)$, that characterises the behaviour of the particles produced. Hence the total field and the total current are given by

$$E(t) = E_{in}(t) + E_{ex}(t) \;, \tag{15}$$

$$j(t) = j_{in}(t) + j_{ex}(t) \;. \tag{16}$$

Continued spontaneous production of charged particle pairs creates a polarisation current, $j_{pol}(t)$, that depends on the particle production rate, $S(\vec{p}, t)$. Meanwhile the motion of the existing particles in the plasma generates a conduction current, $j_{cond}(t)$, that depends on their momentum distribution, $f(\vec{p}, t)$. The internal current is the sum of these two contributions

$$\dot{E}_{in}(t) = -j_{in} = -j_{cond}(t) - j_{pol}(t) \,. \tag{17}$$

At mean field level the currents can be obtained directly from the constraint of local energy density conservation: $\dot{\epsilon} = 0$, where

$$\epsilon(t) = \frac{1}{2}E^2(t) + 2\int \frac{d^3\vec{p}}{(2\pi)^3}\omega(\vec{p}, t) f(\vec{p}, t) \,. \tag{18}$$

For bosons this constraint yields

$$\dot{E}(t) = -2e\int \frac{d^3\vec{p}}{(2\pi)^3}\frac{p_\parallel - eA(t)}{\omega(\vec{p}, t)}\left[f(\vec{p}, t) + \frac{\omega(\vec{p}, t)}{\dot{\omega}(\vec{p}, t)}\frac{df(\vec{p}, t)}{dt}\right], \tag{19}$$

and we can identify the conduction current

$$j_{cond}(t) = 2e \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{p_\parallel - eA(t)}{\omega(\vec{p},t)} f(\vec{p},t) \ , \tag{20}$$

and, using Eq. (2), the polarisation current

$$j_{pol}(t) = \frac{2}{E(t)} \int \frac{d^3p}{(2\pi)^3} \, \omega(\vec{p},t) S(\vec{p},t) \, . \tag{21}$$

Thus, using Eqs. (10) and (11), Maxwell's equation is

$$\dot{E}_{in}(t) = -\ddot{A}_{in}(t) = -2e \int \frac{d^3\vec{P}}{(2\pi)^3} \frac{P_\parallel(t)}{\omega(\vec{P})} \Big[f(\vec{P},t) + \frac{1}{2} v(\vec{P},t) \Big] \, . \tag{22}$$

It is important to observe that this form for the internal field has been employed extensively in the study of back-reactions. However, our contribution is to employ it in conjunction with a time-dependent **external field**, which allows for the exploration of a richer variety of phenomena.

B. Renormalisation

The boson and fermion currents are

$$j_{in}(t) = e g_\pm \int \frac{d^3\vec{P}}{(2\pi)^3} \frac{P_\parallel(t)}{\omega(\vec{P})} \Big[f(\vec{P},t) + \frac{v(\vec{P})}{2} \Big(\frac{\epsilon_\perp}{P_\parallel(t)} \Big)^{g_\pm - 1} \Big] \, , \tag{23}$$

where the integrand depends on the solution of the kinetic equation, Eqs. (12)-(14), which must be such as to ensure the integral is finite. Simple power counting indicates that admissible solutions must satisfy

$$v(\vec{P},t) \, , \; f(\vec{P},t) \overset{|\vec{P}| \to \infty}{\lesssim} \frac{1}{|\vec{P}|^4} \, . \tag{24}$$

To fully characterise the asymptotic behaviour we employ a separable Ansatz

$$f(\vec{P},t) = \sum_{k=0}^{\infty} \frac{f_k(t)}{|\vec{P}|^k} \, , \; v(\vec{P},t) = \sum_{k=0}^{\infty} \frac{v_k(t)}{|\vec{P}|^k} \, , \; z(\vec{P},t) = \sum_{k=0}^{\infty} \frac{z_k(t)}{|\vec{P}|^k} \, . \tag{25}$$

Substituting these in Eqs. (12)-(14) and comparing coefficients, using $P_\parallel \approx \omega(\vec{P}) \approx \epsilon_\perp$, which are valid at large $|\vec{P}|$, we find the leading terms

$$f_4 = \tfrac{1}{16} e^2 E^2(t) \, , \; v_3 = \tfrac{1}{4} e \dot{E}(t) \, , \; z_2 = \tfrac{1}{2} e E(t) \, , \tag{26}$$

with all the lower-order coefficients being zero. Substituting these results in Eq. (20) it is clear that the conduction current is convergent. However, there is a logarithmic divergence in the polarisation current, Eqs. (19) and (21), which is apparent in Eq. (23), but that is just the usual short-distance divergence associated with charge renormalisation. We regularise the polarisation current by writing $v = (v - v_3 P_\parallel/\omega^4) + v_3 P_\parallel/\omega^4$, so that

$$\dot{E}^{\pm}(t) = -j_{ex}(t) \tag{27}$$
$$-g_{\pm} e \int \frac{d^3\vec{P}}{(2\pi)^3} \frac{P_{\|}(t)}{\omega(\vec{P})} \left[f(\vec{P},t) + \tfrac{1}{2} \left\{ v(\vec{P},t) - \frac{e\dot{E}(t)\, P_{\|}(t)}{4\,\omega^4(\vec{P})} \right\} \left(\frac{\epsilon_{\perp}}{P_{\|}(t)} \right)^{g_{\pm}-1} \right] - e^2 \dot{E}^{\pm}(t) I^{\pm}(\Lambda)\,,$$

where

$$I^{\pm}(\Lambda) = \frac{g_{\pm}}{4} \int \frac{d^3\vec{P}}{(2\pi)^3} \frac{P_{\|}^2(t)}{\omega^5(\vec{P})} \left(\frac{\epsilon_{\perp}}{P_{\|}(t)} \right)^{g_{\pm}-1} \stackrel{\Lambda\to\infty}{=} \frac{g_{\pm}}{8\pi^2} \ln\left[\Lambda^2/m^2\right]\,, \tag{28}$$

with Λ a cutoff on $|\vec{P}|$, which effects a regularisation equivalent [9] to Pauli-Villars. Introducing the renormalised charge, fields and current:

$$e_R^2 = Z\, e^2\,,\ \mathcal{E}^{\pm}(t) = E^{\pm}(t)/\sqrt{Z}\,,\ \mathcal{A}^{\pm}(t) = A^{\pm}(t)/\sqrt{Z}\,,\ \mathcal{J}_{ex}(t) = \sqrt{Z}\, j_{ex}(t)\,, \tag{29}$$

with $Z = 1/(1 + e^2\, I^{\pm}(\Lambda))$, and noting that $eE^{\pm}(t) = e_R\, \mathcal{E}^{\pm}(t)$ and $eA^{\pm}(t) = e_R\, \mathcal{A}^{\pm}(t)$, Eq. (27) becomes

$$-\ddot{\mathcal{A}}^{\pm}(t) = \dot{\mathcal{E}}^{\pm}(t) = -\mathcal{J}_{ex}(t) \tag{30}$$
$$-g_{\pm} e_R \int \frac{d^3\vec{P}}{(2\pi)^3} \frac{P_{\|}(t)}{\omega(\vec{P})} \left[f_{\pm}(\vec{P},t) + \tfrac{1}{2} \left\{ v_{\pm}(\vec{P},t) - \frac{e_R\, \dot{\mathcal{E}}^{\pm}(t)\, P_{\|}(t)}{4\,\omega^4(\vec{P})} \right\} \left(\frac{\epsilon_{\perp}}{P_{\|}(t)} \right)^{g_{\pm}-1} \right]\,.$$

This defines a properly renormalised equation for the fields. Our procedure is technically different from that employed elsewhere [9,18] but yields an equivalent result. Subsequently all fields and charges are to be understood as renormalised.

C. Numerical results

Equations (12)-(14) together with Maxwell's equation, Eq. (27), form a coupled system of differential equations. To solve it we first evaluate the internal current from Eq. (27) at the primary time-slice using the initial conditions for the distribution function. That, via Eqs. (15) and (16), provides an electric field, which we use to calculate the momentum distribution from Eqs. (12)-(14). This procedure is repeated as we advance over our time-grid. We use a momentum grid with 200 transverse- and 400 longitudinal-points, a time-step $dt = 0.005$, and $\Lambda = 50$ in Eq. (28). All dimensioned quantities are expressed in units of m, the parton mass.

Spontaneous particle creation occurs in the presence of a strong field under whose influence the vacuum becomes unstable and decays. Herein we induce this by a time-dependent external field and compare three different field configurations. The fields vanish at $t \to -\infty$, and at $t = t_0$ the magnitude of the field increases and eventually leads to particle creation. Configuration (i): For comparison with Refs. [8,9,14], we solve the set of equations as an initial value problem without an external field, using an initial value of the electric field that is large enough to cause pair production. Configuration (ii): We employ

$$A_{ex} = -A_0 b^2\, [t/b + \ln(2\cosh(t/b))], \qquad E_{ex}(t) = A_0 b\, [\tanh(t/b) + 1]\,, \tag{31}$$

which is an electric field that "switches-on" at $t \sim -b$ and evolves to a constant value, $2A_0 b$, in an interval $t \sim 2/b$. Configuration (iii): Is an impulse field configuration:

$$A_{ex}(t) = A_0[\tanh(t/b) + 1], \qquad E_{ex}(t) = -A_0[b\cosh^2(t/b)]^{-1}, \tag{32}$$

which is an electric field that "switches-on" at $t \sim -2b$ and off at $t \sim 2b$, with a maximum magnitude of A_0/b at $t = 0$. Once this field has vanished only the induced internal field remains to create particles and affect their motion.

For configuration (i) we fix an initial value of $E(t = 0) = 10$, in units of m, with $e^2 = 4$, and for bosons obtain the electric field and current depicted in Fig. 1, where plasma oscillations are evident. The frequency of these oscillations increases with the magnitude of the field. The current exhibits a plateau for small t, when the particles reach their maximum velocity. This current opposes the field, and leads to a suppression of particle production and a deceleration of the existing particles. The effect of this is to overwhelm the field and change its sign with a consequent change in the direction of the particles' collective motion. The process repeats itself, yielding the subsequent oscillations that persist in the absence of additional interactions, such as collisions or radiation. The structure visible at the peaks and troughs of the current is **not** a numerical artefact. It is related to the field-strength/mass ratio, being more pronounced for large values, and occurs on a time-scale $\sim \tau_{qu}$, the vacuum tunnelling time, and hence can be characterised as *Zitterbewegung*. It disappears if an ideal-Markovian approximation to the source term is used because that cannot follow oscillations on such small time-scales [14]. The t-dependence of the $\vec{p} = 0$ distribution function is depicted in Fig. 2, where the beat-like pattern is the result of back-reactions and the rapid fluctuations coincide with the *Zitterbewegung* identified in the current.

Configurations (ii) and (iii) are alike in that the field "switches-on" at a given time. However, for (ii) the external field remains constant as t increases whereas in (iii) it "switches-off" after $t \sim 2/b$. The electric field and current obtained for bosons in these cases are depicted in Figs. 3 and 4: plasma oscillations are again evident. We plot the **total** electric field and thus it is evident in Fig. 3 that the internal electric field evolves to completely compensate for the persistent external field, which alone would appear as a straight-line at $E(t) = 7$. Unsurprisingly, as we see from Fig. 4, a stable state is reached more quickly in the absence of a persistent electric field. Outside the temporal domain on which the vector potential acts, the initial value and impulse solutions are equivalent.

We illustrate the results for fermions in Fig. 5 using the impulse configuration. The amplitude and frequency of the plasma oscillations are significantly larger than for bosons in a configuration of equal strength. Further, the stable state is reached more quickly because Pauli blocking inhibits particle production; i.e., no particles can be produced once all available momentum states are occupied. Pauli blocking also guarantees $f_-(\vec{p}, t) < 1$, for all t.

We have also calculated the $P_\parallel$- and $p_\perp$-dependence of f for both bosons and fermions. We find $f_+(t = 0) = 0$; i.e., bosons cannot be produced with zero kinetic momentum, an effect readily anticipated from Eq. (6). For small t, $f_\pm(\vec{p}, t)$ is a slowly varying function of $\vec{p}$ on its domain of support. However, with increasing t, $f_\pm(\vec{p}, t)$ develops large-magnitude fluctuations without increasing that domain. The momentum-space position of the midpoint of the domain of support oscillates with a t-dependence given by the kinetic momentum: $P_\parallel = p_\parallel - eA(t)$.

One additional observation is important here. The magnitude, $\propto A_0$, of the electric fields we have considered is large and hence the time between pair production events, τ_{prod}, is small, being inversely proportional to the time-average of the source term, S. The period

of the plasma oscillations, τ_{pl}, also decreases with increasing A_0 but nevertheless we always have $\tau_{prod} \ll \tau_{pl}$. Thus, in contrast to the effect it has on the production process [17], the temporal nonlocality of the non-Markovian source term is unimportant to the collective plasma oscillation.

D. Collisions

In the previous subsection we ignored the effect of collisions when treating the spontaneous production of charged particles and subsequent evolution of the plasma. Now we consider the effect of a simple collision term [21]

$$C_\pm(\vec{p},t) = \frac{f^{eq}_\pm(\vec{p},t) - f_\pm(\vec{p},t)}{\tau_r}\,, \tag{33}$$

where τ_r is the "relaxation time" and $f^{eq}_\pm$ are the thermal equilibrium distribution functions for bosons and fermions:

$$f^{eq}_\pm(\vec{p},t) = \frac{1}{\exp[\omega(\vec{p},t)/T(t)] \mp 1}\,. \tag{34}$$

Here $T(t)$ is the "instantaneous temperature", which is a model-dependent concept [5,19,20], and since our results are not particularly sensitive to details of its form we employ a simple parametrisation

$$T(t) = T_{eq} + (T_m - T_{eq})\,\mathrm{e}^{-t^2/t_0^2}\,, \tag{35}$$

with an equilibrium temperature $T_{eq} = 1.0$, a maximum temperature $T_m = 2.0$, and a profile-width $t_0^2 = 10 \sim \tau_{pl}$. The collision term is added to the r.h.s. of Eq.(2), which becomes

$$\frac{df_\pm(\vec{p},t)}{dt} = S_\pm(\vec{p},t) + C_\pm(\vec{p},t)\,. \tag{36}$$

This "relaxation time" approximation assumes that the system evolves rapidly towards thermal equilibrium after the particles are produced. It has been used before, both in the absence [4] of back-reactions and including them [12,19,20], but with source terms that neglect fluctuations on short time-scales.

We note that in the low density limit: $f(\vec{p},t) \ll 1$, one can neglect the distribution function in the source term, Eq. (3), and Eq. (36) has the simple solution

$$f^0_\pm(\vec{p},t) = \int_{-\infty}^{t} dt' \exp\left[\frac{t'-t}{\tau_r}\right]\left(S^0_\pm(\vec{p},t') + \frac{f^{eq}_\pm(\vec{p},t')}{\tau_r}\right)\,. \tag{37}$$

In our numerical studies we treat τ_r as a parameter and study the effect of C on the plasma oscillations. Our results for bosons using this crude approximation are depicted in Fig. 6. For $\tau_r \gg \tau_{pl}$ the oscillations are unaffected, as anticipated if $1/\tau_r$ is interpreted as a collision frequency. For $\tau_r \sim \tau_{pl}$ the collision term has a significant impact, with both the amplitude and frequency of the plasma oscillations being damped. There is a τ_r below which no oscillations arise and the systems evolves quickly and directly to thermal equilibrium.

IV. SUMMARY AND CONCLUSION

We have studied spontaneous particle creation in the presence of back-reactions and collisions, both of which dramatically affect the solution of the kinetic equation. The back-reactions lead to plasma oscillations that are damped by the thermalising collisions if the collision frequency is comparable to the plasma frequency. In electric fields where the period of the plasma oscillations is large compared to the time-scales characterising particle production, the non-Markovian features of the source term play little role in the back-reaction process.

Plasma oscillations are a necessary feature of all studies such as ours but are they relevant to the creation of a quark-gluon plasma? If we set the scale in our calculations by assuming that fermions are created in the impulse configuration with $\langle \epsilon_\perp \rangle \sim \Lambda_{\rm QCD} \sim 0.5\sqrt{\sigma}$, the QCD string tension, then $A_0 = 10$ with $e^2 = 5$ corresponds to an initial field strength $eE \sim 15\,\sigma$ and energy density $\frac{1}{2}E^2 \sim 20\,\sigma^2$. These are very large values but even so the plasma oscillation period is still large: $\tau_{pl} = 5\,{\rm fm/c}$, and collisions can only act to increase that. We therefore expect that a quark-gluon plasma will have formed and decayed well before plasma oscillations can arise. On these short time-scales non-Markovian effects will be important.

Our estimate shows back-reactions to be unimportant on small time-scales but that is not true of collisions. However, it is clear that in QCD applications they must be described by something more sophisticated than the "relaxation-time" approximation.

Finally, with $1/\Lambda_{\rm QCD}$ setting the natural scale, the finite interaction volume will clearly be important and hence the assumption of a spatially homogeneous background field must also be improved before calculations such as ours are relevant to a quark-gluon plasma. Ref. [22] is a step in that direction.

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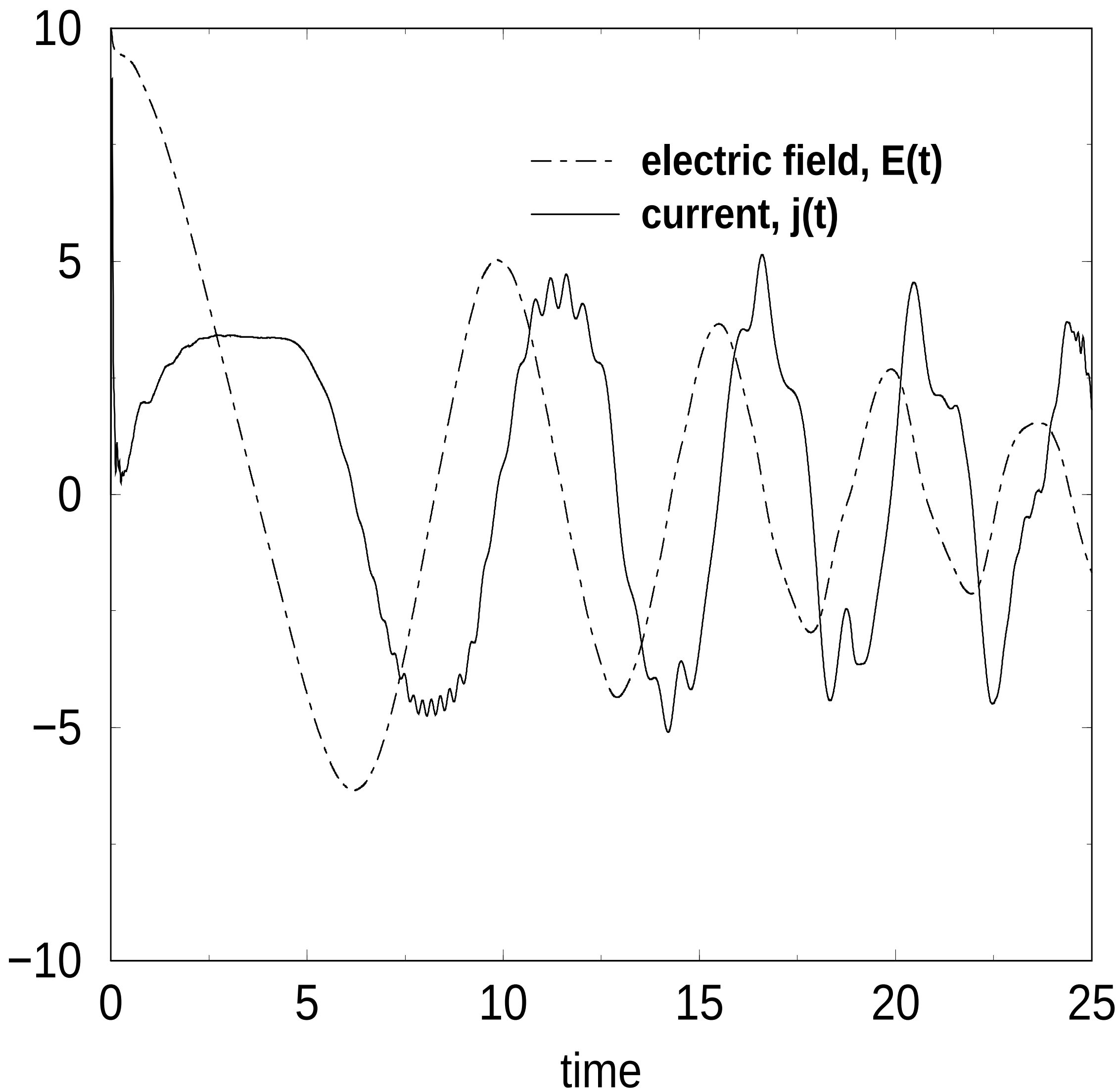

FIG. 1. Time evolution for bosons of the total electric field and total current with initial values $E = 10$ and $e^2 = 4$. (Here, as in the text, all dimensioned quantities are given in units of the mass-scale m.)

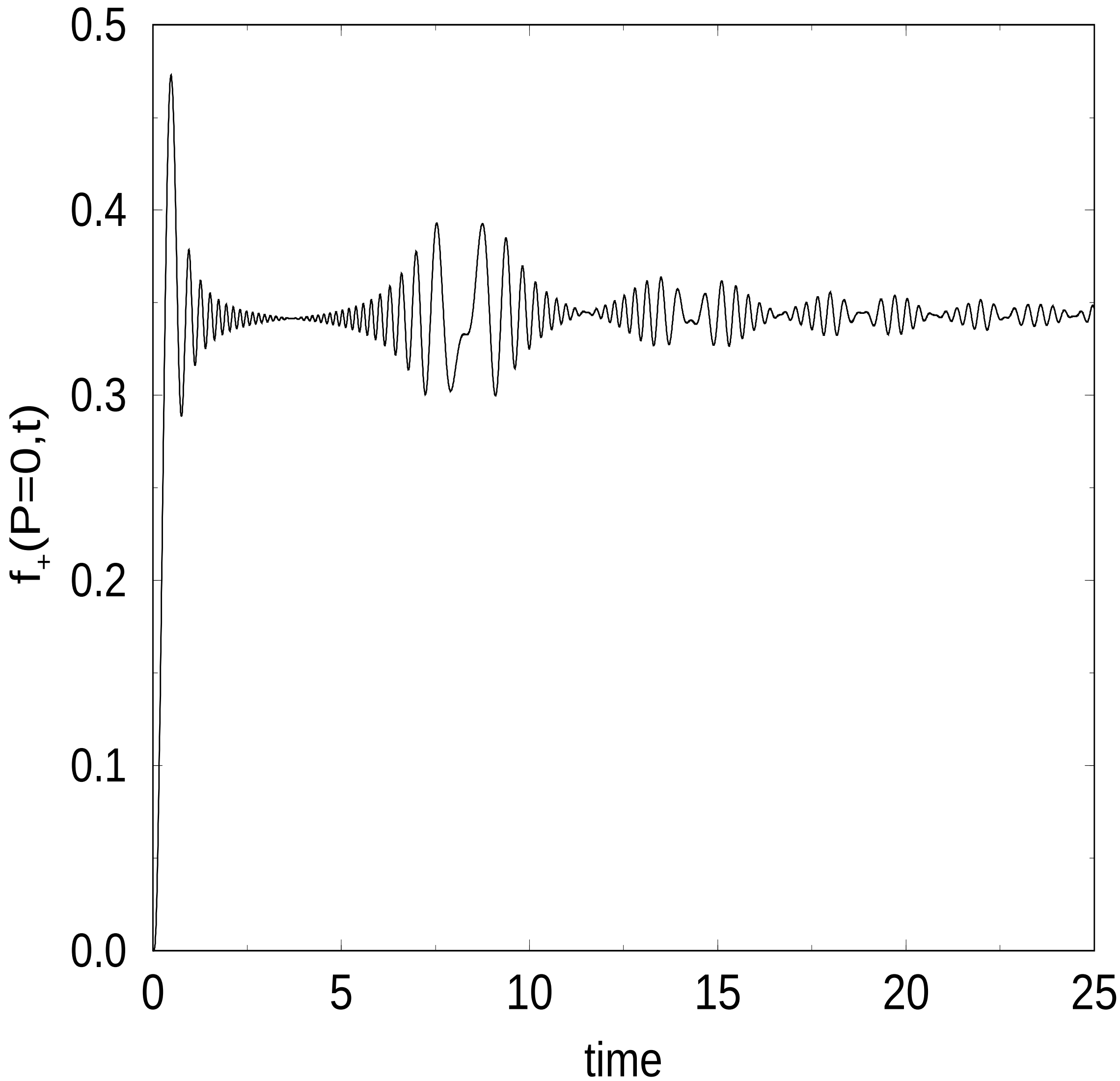

FIG. 2. Time evolution of the distribution function $f(\bar{0}, t)$ with initial values $E = 10$ and $e^2 = 4$ for bosons. The rapid fluctuations occur on the vacuum tunnelling time-scale, τ_{qu}, and coincide with the *Zitterbewegung* identified in the current. In the absence of back-reactions these recurrent fluctuation packets are absent [17].

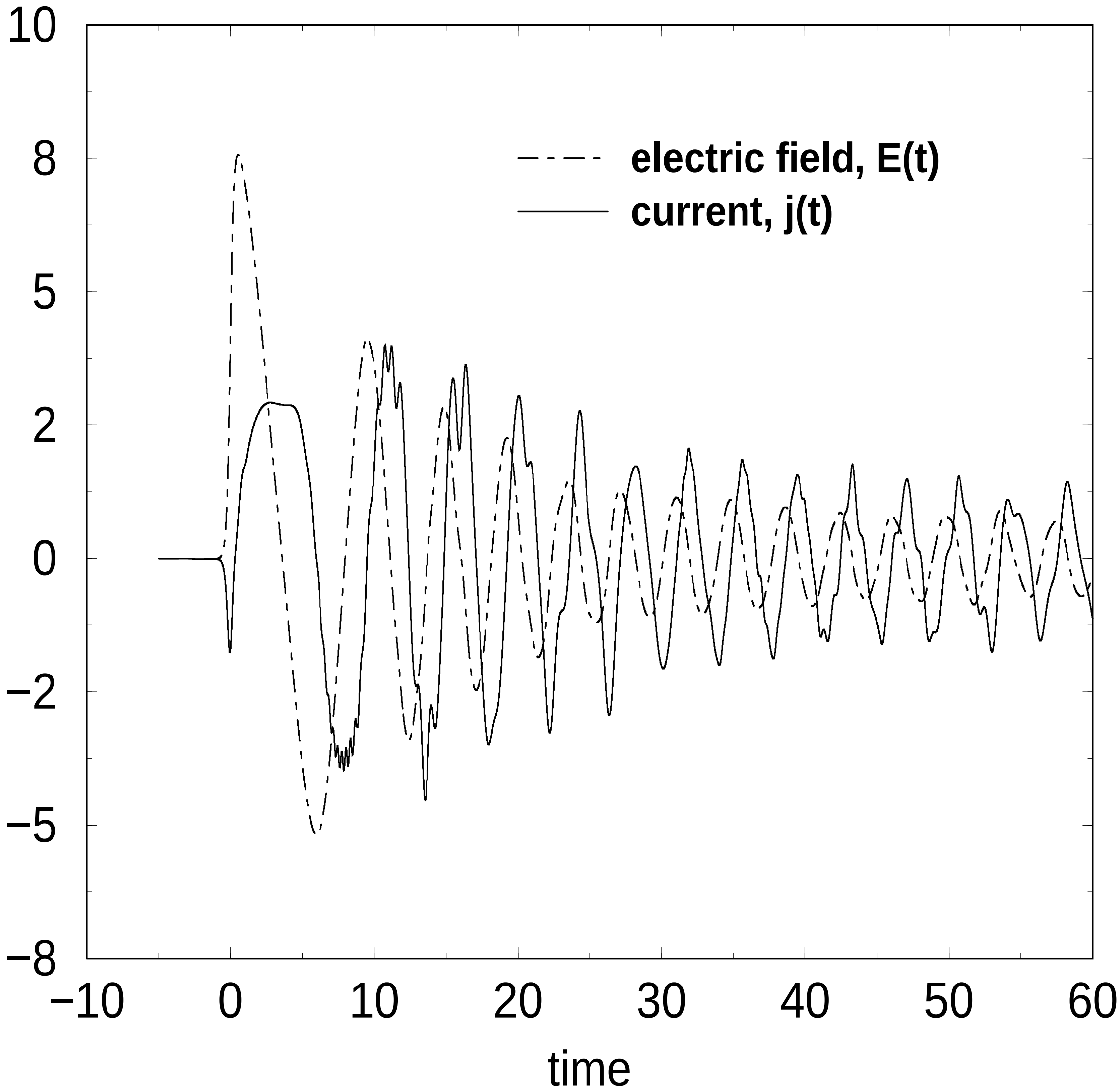

FIG. 3. Time evolution for bosons of the electric field and the current for the step function external field, Eq. (31), with $A_0 = 14.0$, $b = 0.25$ and the coupling $e^2 = 6$.

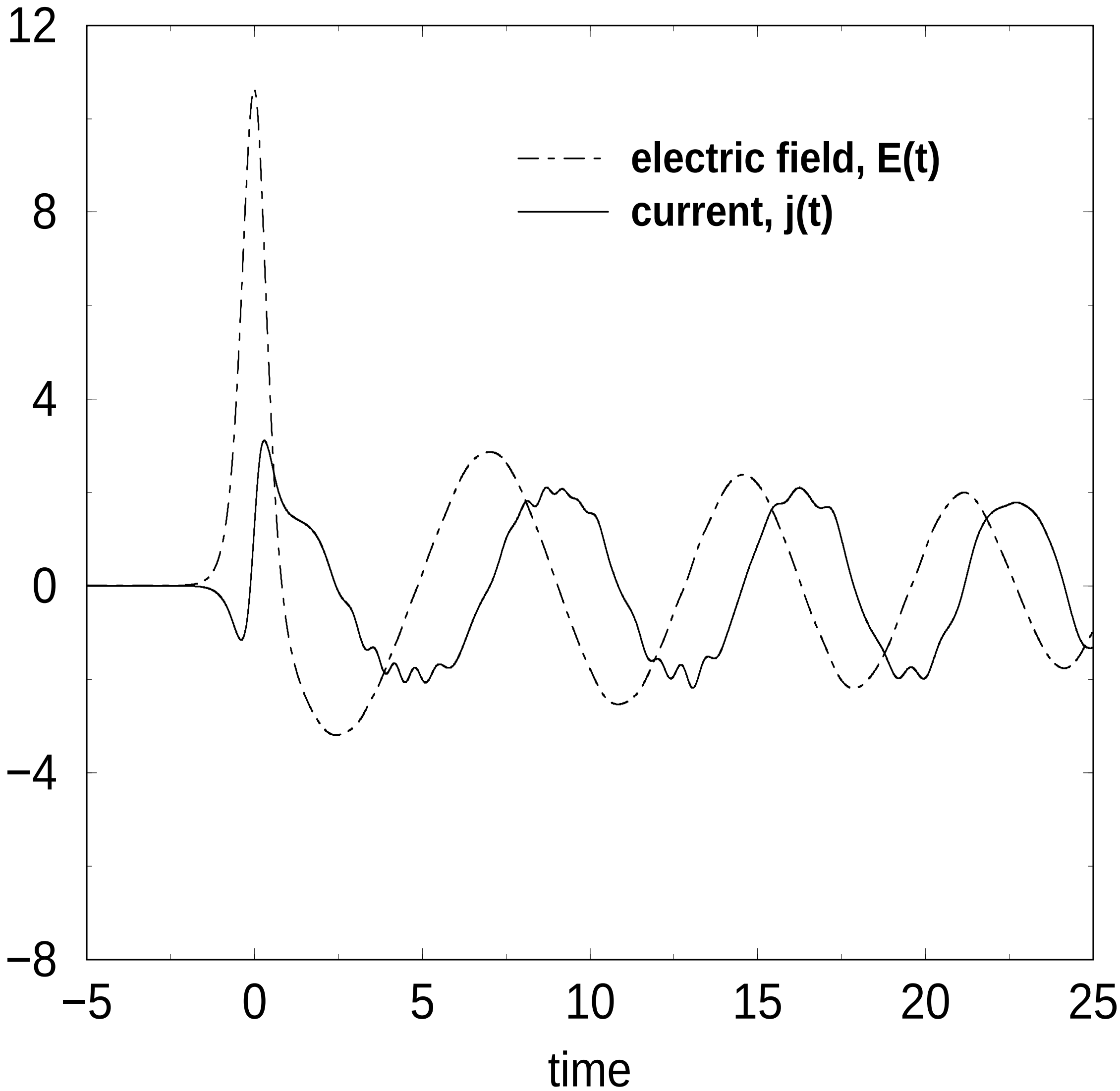

FIG. 4. Time evolution for bosons of the electric field and current for an impulse external field, Eq. (32), with $A_0 = 10.0$, $b = 0.5$ and the coupling $e^2 = 5$.

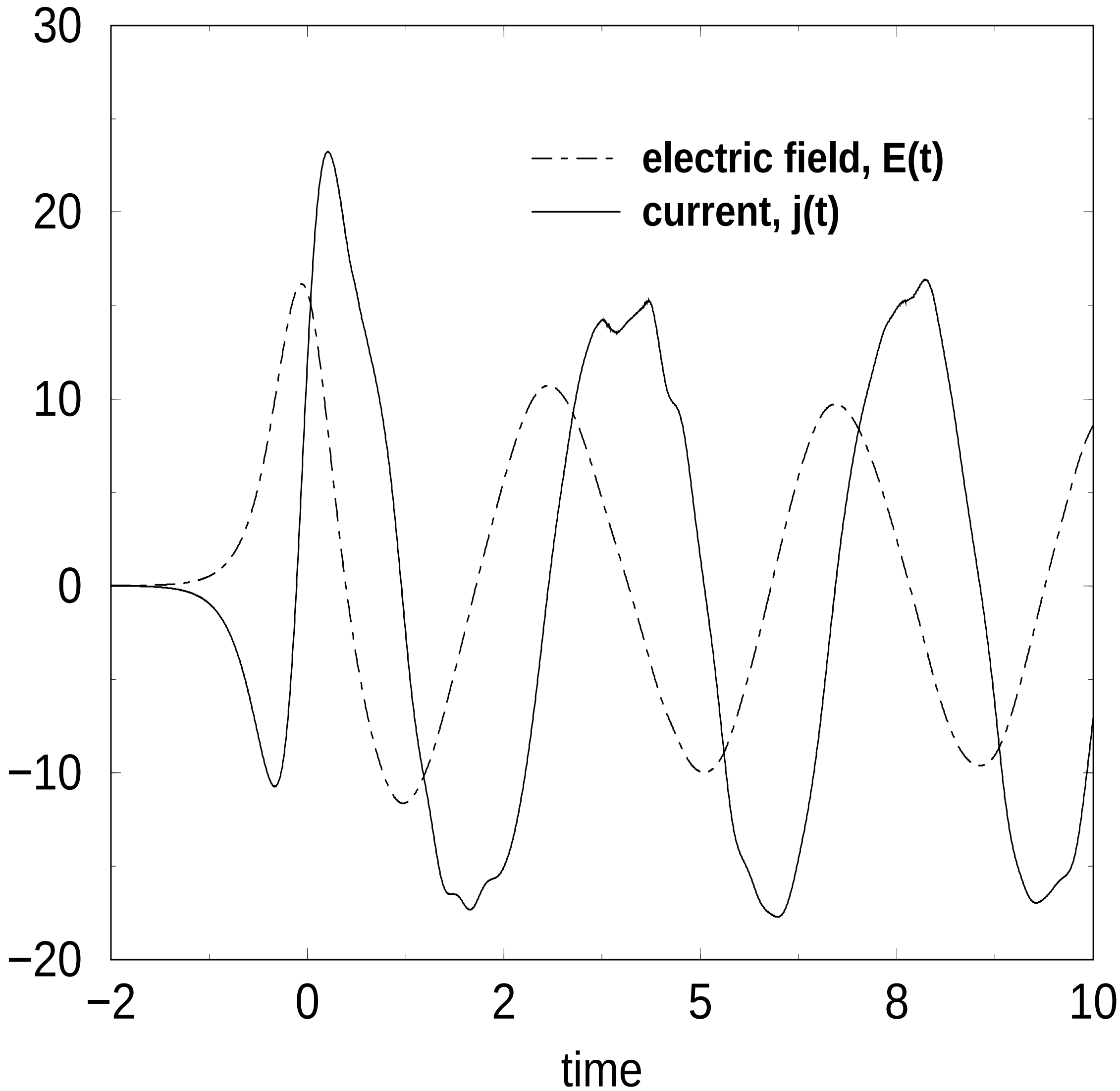

FIG. 5. Time evolution for fermions of the electric field and current for an impulse external field, Eq. (32), with $A_0 = 10.0$, $b = 0.5$ and the coupling $e^2 = 4$.

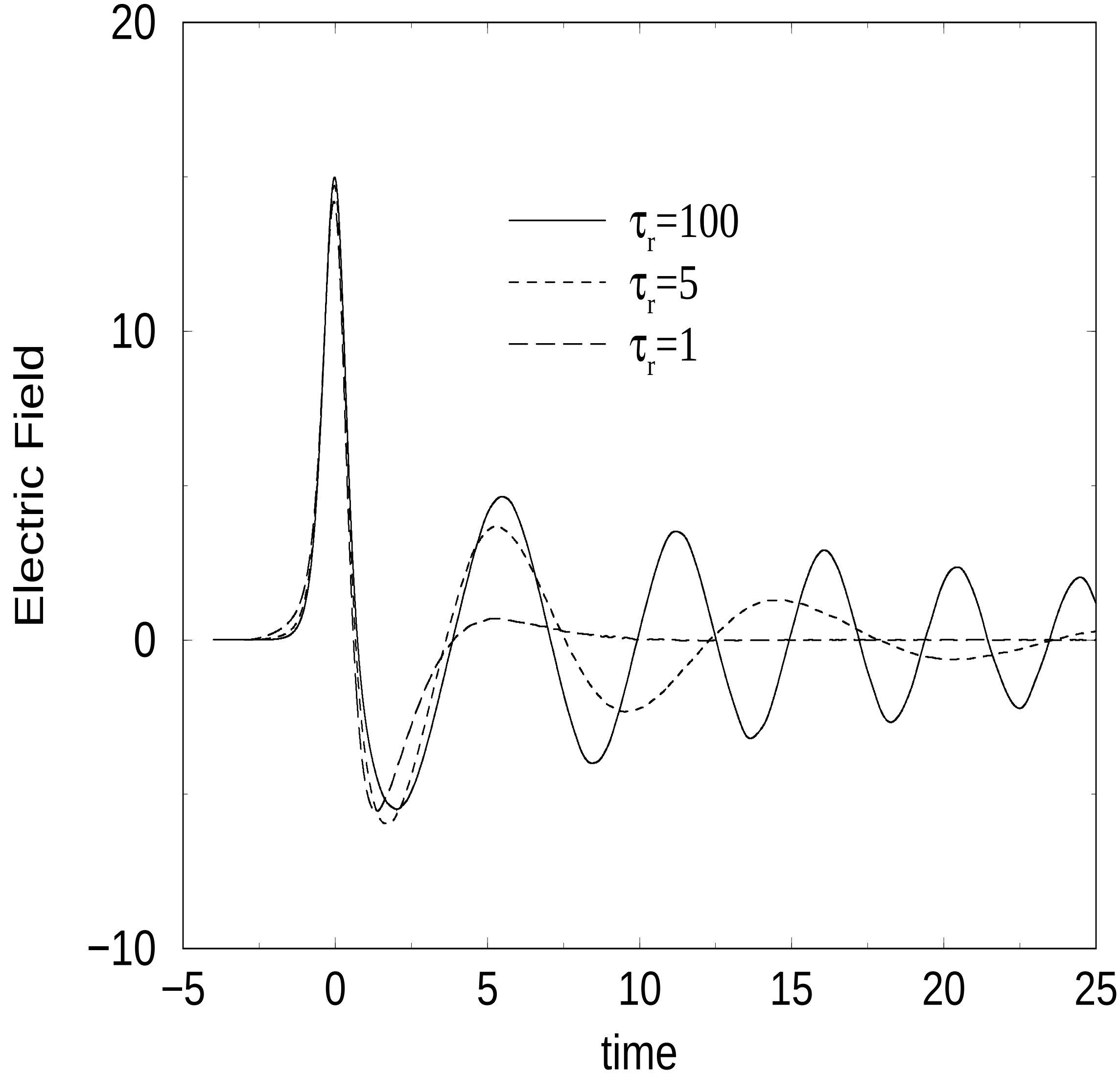

FIG. 6. Time evolution for bosons of the electric field obtained using different relaxation times in the collision term of Eq. (33), and with the impulse external field, Eq. (32), where $A_0 = 7.0$, $b = 0.5$ and the coupling $e^2 = 4$.